

Grab a worksheet from the front table and start warming up!

Classwork - NEW UNIT! (Linear Relationships) Constant Rate of Change

Example of finding rate of change on tables.

IU-x DU-y

Cars Washed	
Number	Money (\$)
5	40
10	80
15	120
20	160

+5C +40
 +5C +40
 +5C +40

Find the unit rate to determine the constant rate of change.

$$\frac{\text{change in money}}{\text{change in cars}} = \frac{40 \text{ dollars}}{5 \text{ cars}}$$

The money earned increases by \$40 for every 5 cars.

$$= \frac{8 \text{ dollars}}{1 \text{ car}}$$

Write as a unit rate.

So, the number of dollars earned increases by \$8 for every car washed.

Find the constant rate of change for each table. **SHOW WORK** and describe your answer.

1.

Time Spent Mowing (h)	Money Earned (\$)
1	10
3	30
5	50
7	70

+2 +20
 +2 +20
 +2 +20

$\frac{\$20}{2h}$

Rate of change = \$10/h

2.

Number of Trees	Number of Apples
5	100
10	200
15	300
20	400

+5 +100
 +5 +100
 +5 +100

$\frac{100 \text{ apples}}{5 \text{ trees}}$

Rate of change = 20 apples/tree

3.

Time (min)	Altitude (m)
2	650
3	575
5	425
8	200

+1
+2
+3

-75
-150
-225

$$\frac{-75m}{1min} = -75m/min$$

$$\frac{-150m}{2min} = -75m/min$$

$$\frac{-225m}{3min} = -75m/min$$

Rate of change = -75m/min

Example of finding rate of change on graphs.

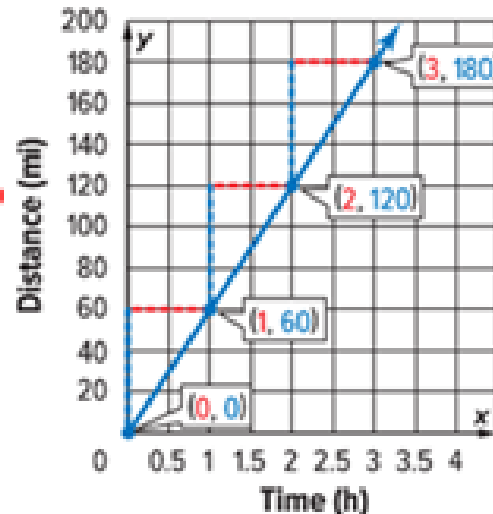
The graph represents the distance traveled while driving on a highway. Find the constant rate of change.

To find the rate of change, pick any two points on the line, such as (0, 0) and (1, 60).

$$\frac{\text{change in miles}}{\text{change in hours}} = \frac{(60 - 0) \text{ miles}}{(1 - 0) \text{ hours}}$$

$$= \frac{60 \text{ miles}}{1 \text{ hour}}$$

DU



DU

You can also transfer the data from the graph to a table to find the rate of change.

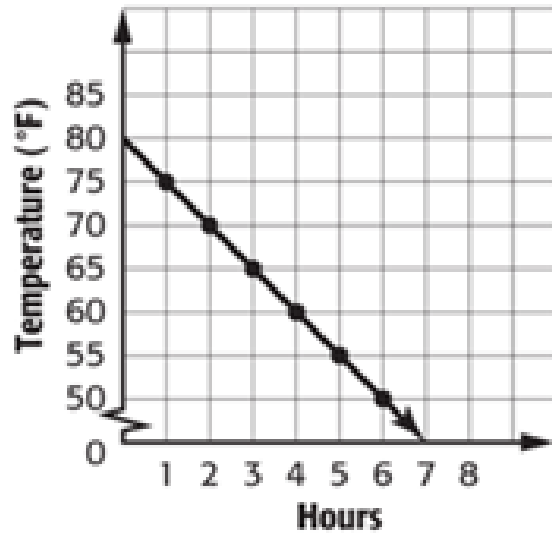
DU-x *DU-y*

Time (min)	Distance (mi)
0	0
1	60
2	120
3	180

Find the constant rate of change for each graph.

4.

Temperature Change



Hours	Temperature
1	75
2	70
3	65
6	50

+1
+1
+3

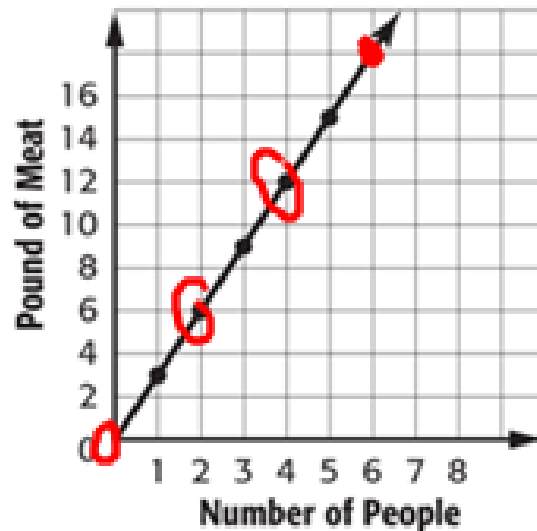
-5
-5
-15

$$\frac{-15^{\circ}\text{F}}{3\text{h}}$$

Rate of change = -5°F/h

5.

Meat Consumption



# of People	lbs. of Meat
0	0
2	6
4	12
6	18

+2
+2
+2

+6
+6
+6

$$\frac{6\text{lbs}}{2\text{ People}}$$

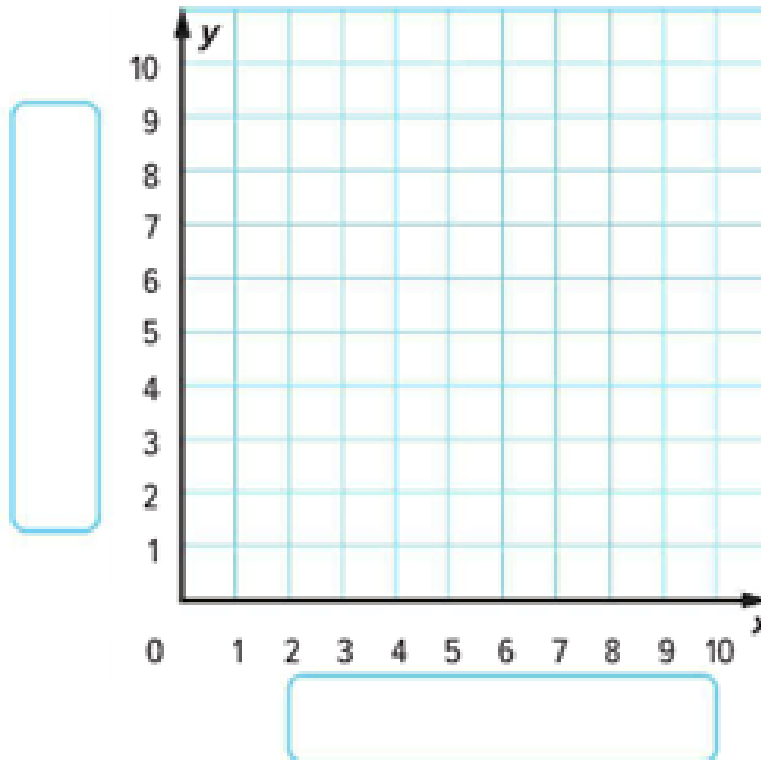
Rate of change = 3lbs/Person

Music Marcus can download two songs from the Internet each minute. This is shown in the table below.

Time (minutes), x	0	1	2	3	4
Number of Songs, y	0	2	4	6	8

1. Compare the change in the number of songs y to the change in time x . What is the rate of change?

2. Graph the ordered pairs from the table on the graph shown. Label the axes. Then describe the pattern shown on the graph.



Linear Relationships

Relationships that have straight-line graphs, like the one on the previous page, are called **linear relationships**. Notice that as the number of songs increases by 2, the time in minutes increases by 1.

Number of Songs, y	0	2	4	6	8
Time (minutes), x	0	1	2	3	4

Blue arrows above the table indicate an increase of +2 in the number of songs between each step. Red arrows below the table indicate an increase of +1 in time between each step.

Rate of Change
 $\frac{2}{1} = 2$ songs per minute

The rate of change between any two points in a linear relationship is the same or *constant*. A linear relationship has a **constant rate of change**.

Example



1. The balance in an account after several transactions is shown. Is the relationship between the balance and number of transactions linear? If so, find the constant rate of change. If not, explain your reasoning.

Number of Transactions	Balance (\$)
3	170
6	140
9	110
12	80

Red arrows on the left indicate an increase of +3 in the number of transactions between each step. Blue arrows on the right indicate a decrease of -30 in the balance between each step.

As the number of transactions increases by 3, the balance in the account decreases by \$30.

Since the rate of change is constant, this is a linear relationship. The constant rate of change is $\frac{-30}{3}$ or $-\$10$ per transaction. This means that each transaction involved a \$10 *withdrawal*.

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Got it? Do these problems to find out

a.

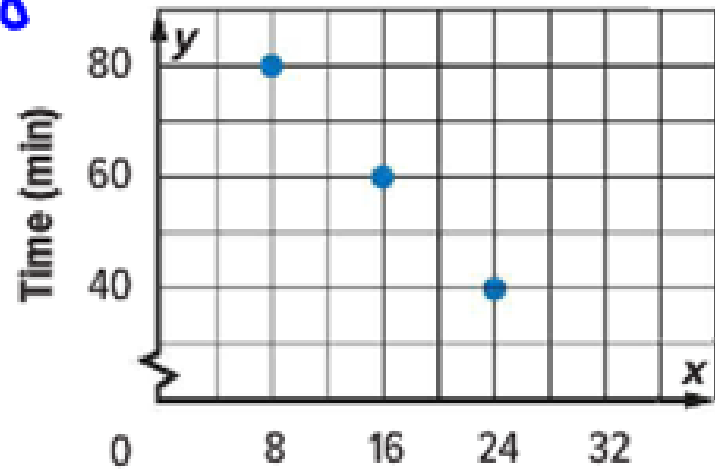
Cooling Water	
Time (min)	Temperature (°F)
5	95
10	90
15	86
20	82

↓ ↓ ↓ ↓

↑ ↓ ↓ ↓

b.

DV



Number of Volunteers

I U

$$\frac{dy}{dx} = -2.5 \text{ min/volunteer}$$

$$\frac{-5^{\circ}\text{F}}{5\text{min}} = -1^{\circ}\text{F/min}$$

$$\frac{-4^{\circ}\text{F}}{5\text{min}} = -0.8^{\circ}\text{F/min}$$

Not Linear

# of Vol.	Time (min)
8	80
16	60 - 20
24	40 - 20

Proportional Linear Relationships

Words Two quantities a and b have a proportional linear relationship if they have a constant ratio and a constant rate of change.

Symbols $\frac{b}{a}$ is constant and $\frac{\text{change in } b}{\text{change in } a}$ is constant.

To determine if two quantities are proportional, compare the ratio $\frac{b}{a}$ for several pairs of points to determine if there is a constant ratio.

2. Use the table to determine if there is a proportional linear relationship between a temperature in degrees Fahrenheit and a temperature in degrees Celsius. Explain your reasoning.

Degrees Celsius	0	5	10	15	20
Degrees Fahrenheit	32	41	50	59	68

+5 +5 +5 +5
+9 +9 +9 +9

Constant Rate of Change

$$\frac{\text{change in } ^\circ\text{F}}{\text{change in } ^\circ\text{C}} = \frac{9}{5}$$

Since the rate of change is constant, this is a linear relationship.

To determine if the two scales are proportional, express the relationship between the degrees for several columns as a ratio.

$$\frac{\text{degrees Fahrenheit}}{\text{degrees Celsius}} \rightarrow \frac{41}{5} = 8.2 \quad \frac{50}{10} = 5 \quad \frac{59}{15} \approx 3.9$$

Since the ratios are not the same, the relationship between degrees Fahrenheit degrees Celsius is *not* proportional.

Check: Graph the points on the coordinate plane. Then connect them with a line.

The points appear to fall in a straight line so the relationship is linear. ✓

The line connecting the points does not pass through the origin so the relationship is not proportional. ✓

