Get out your homework and turn to page 242 and Warm Up on #21 and 22.

Classwork - Solving Systems of Equations Algebraically

21. Determine if each system of equations has no solution, one solution, or infinitely many solutions. Select the correct answer.

a. $y = 3x - 1$ ☐ no solution   ☒ one solution   ☐ infinitely many solutions
$y = -2x + 4$

b. $y = 4x - 2$ ☒ no solution   ☐ one solution   ☐ infinitely many solutions
$y = 4x + 5$

c. $y = -x - 3$ ☐ no solution   ☒ one solution   ☐ infinitely many solutions
$y = x$

22. Yolanda scored 3 times as many goals as Xander. Xander scored 4 fewer goals than Yolanda. The number of goals scored by each person can be represented by the system of equations below.
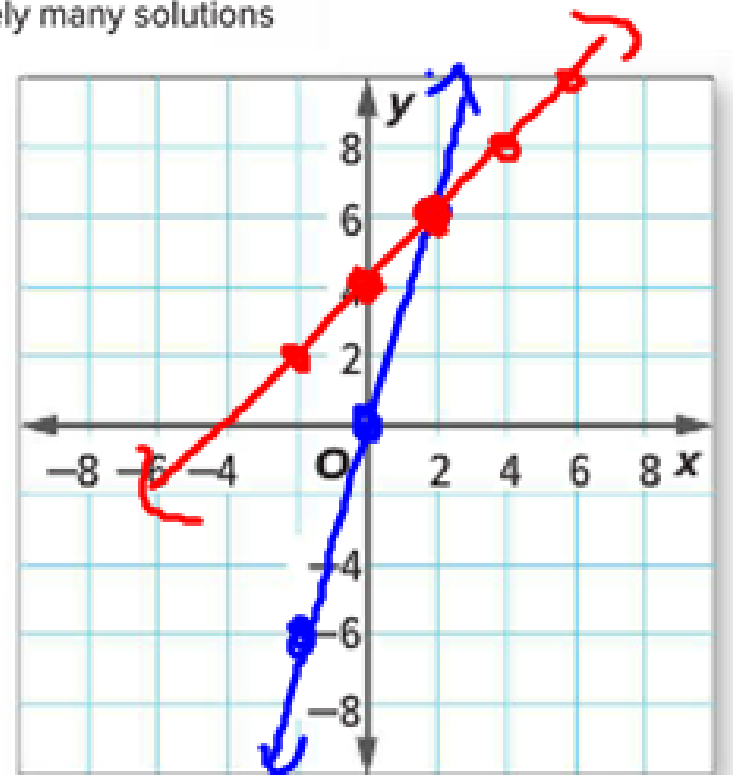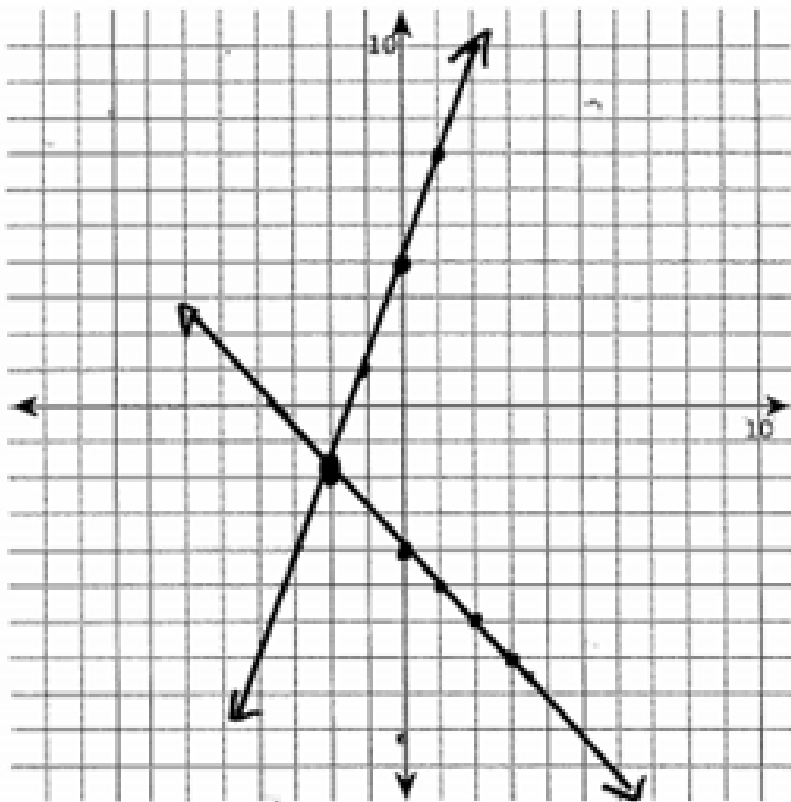
$y = 3x$

$y = x + 4$

Graph these equations on the coordinate plane.

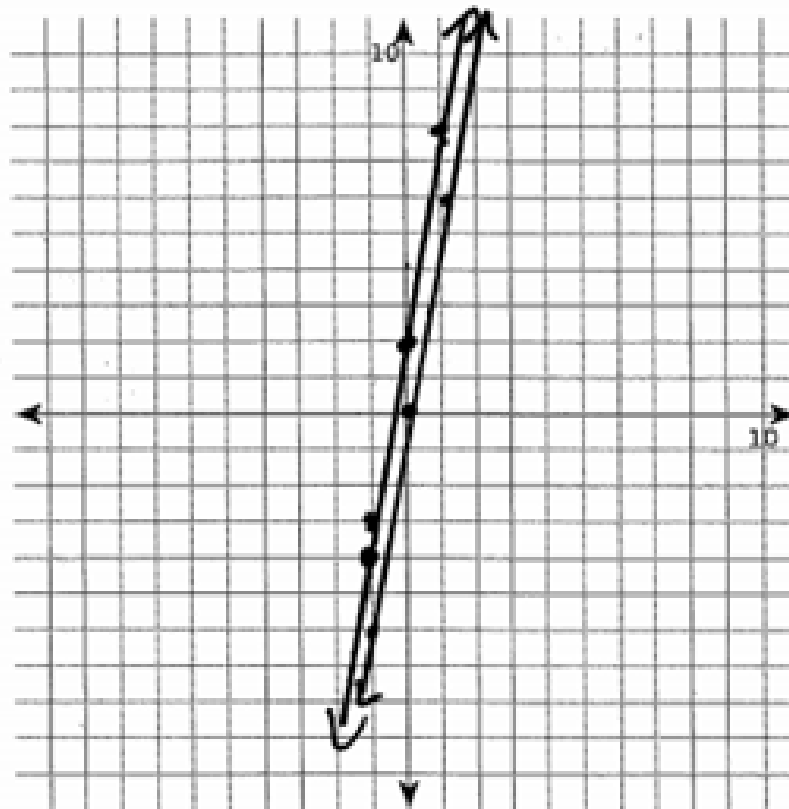What is the solution to the system of equations? What does this situation represent?
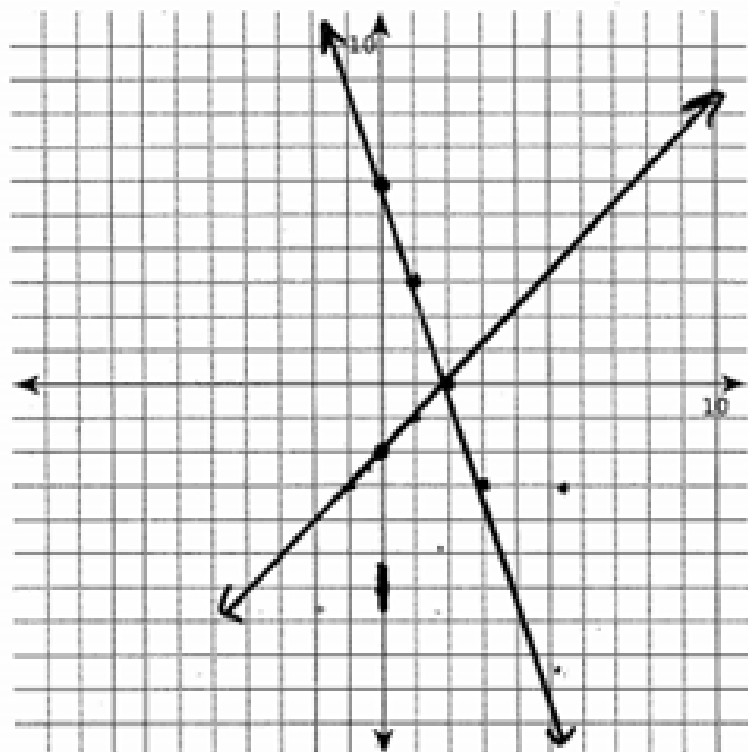
$(2, 6)$

1. $y = 3x + 4$,
   $y = -x - 4$

2. $y = 6x + 2$ same slope
   $y = 6x$

# of Solution(s): ___1___   Solution: $(-2, -2)$

# of Solution(s): ___O___   No Solutions
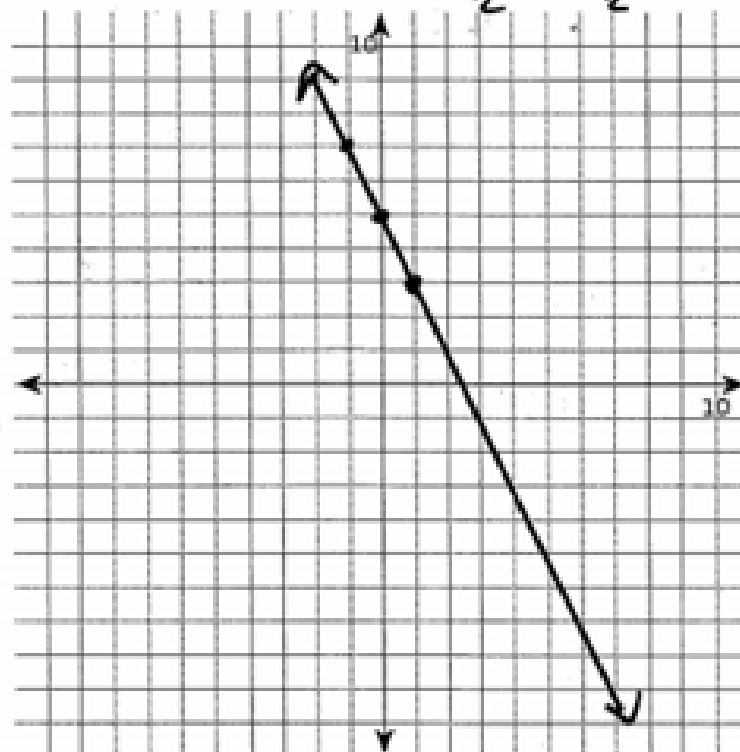Solution: (,   )

3. $y = -3x + 6$
   $y = x - 2$

# of Solution(s): _____1_____   Solution: (2 , 0)

4. $y = -2x + 5$
   $4x + 2y = 10$

$4x + 2y = 10$
$-4x \quad -4x$
$\dfrac{2y}{2} = \dfrac{-4x + 10}{2}$ $y = -2x + 5$

# of Solution(s): _____∞_____   Solution: (All Real Numbers ,)

5. BASKETBALL Alonzo and Miguel scored a total of 48 points in the basketball game. Miguel scored 16 more points than Alonzo.

$y \to m =$ Miguel

$x \to r =$ Alonzo

$x + y = 48$

Equation 1 → $m + r = 48$

Equation 2 → $m = r + 16$
$y = x + 16$

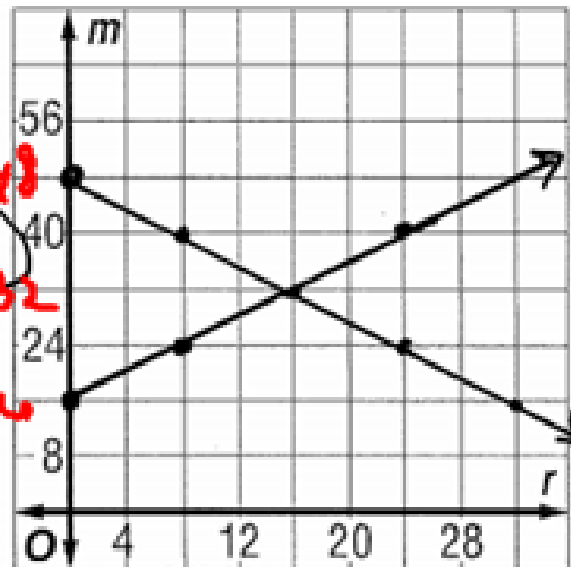Solution = $(16, 32)$
Alonzo scored 16 points
Miguel scored 32 points

$m + r = 48$
$\underline{-r \quad -r}$
$m = -r + 48$
slope $= -1 = \frac{-8}{8}$
y-int $= 48$

48
32
16

6. ANIMALS The total number of cats and dogs at the shelter is 110. There are 10 more cats than dogs.

$(y) c =$ cats

$(x) d =$ dogs

$(50, 60)$
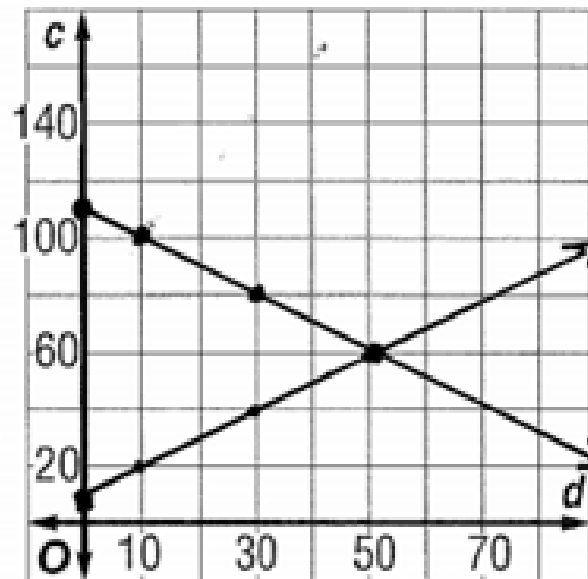The are 50 dogs and 60 cats

$y + x = 110$ $y = -x + 110$

Equation 1 → $c + d = 110 \to c = -d + 110$

Equation 2 → $c = d + 10$

7. PING-PONG Jenny won the ping-pong championship eight more times than Gerardo. They have won a combined total of 32 championships.
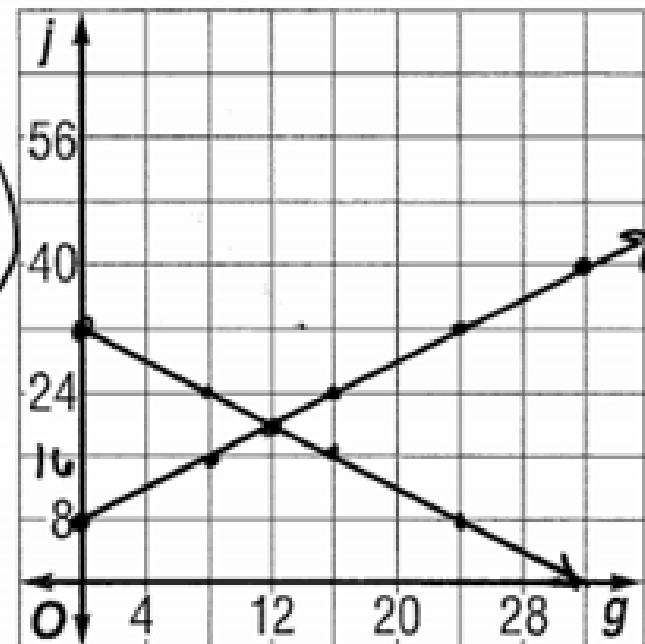
$(y)\ j = $ Jenny

$(x)\ g = $ Gerardo

$(12, 20)$

Gerardo won $12 and Jenny won 20

Equation 1 → $j = g + 8$ $y = x + 8$

Equation 2 → $j + g = 32$ $y + x = 32$

$$\underline{\qquad -g \quad -g \qquad}$$

$$j = -g + 32$$

Real-World Link

Jewelry Mary Anne sold 20 necklaces and bracelets at the craft fair. She sold 3 times as many necklaces as bracelets.

Step 1 The bar diagram below represents the situation

x ▭
bracelets

y ▭
necklaces

} 20

An equation to represent the bar diagram is $x + y = 20$.

Step 2 Mary Anne sold 3 times as many necklaces as bracelets. Divide the necklace bar into sections to represent this.

x ▭
bracelets

$y = 3x$ ▭
necklaces

} 20

Write an equation using only x to represent the total number of necklaces and bracelets.

Step 3 Solve the equation from Step 2. What does the solution represent? _____

1. How many bracelets and necklaces did Mary Anne sell?

▭ bracelets and ▭ necklaces

In the previous lesson, you estimated the solution of a system of equations by graphing. **Substitution** is an algebraic model that can be used to find the exact solution of a system of equations.

Example

1. Solve the system of equations algebraically.

$$y = x - 3$$
$$y = 2x$$

Since y is equal to $2x$, you can replace y with $2x$ in the first equation.

$y =$	$x - 3$	Write the equation.
$2x =$	$x - 3$	Replace y with $2x$.
$-x = -x$		Subtraction Property of Equality
$x = -3$		Simplify.

Since $x = -3$ and $y = 2x$, then $y = -6$ when $x = -3$. The solution of this system of equations is $(-3, -6)$.

Check Graph the system.

Handwritten work:
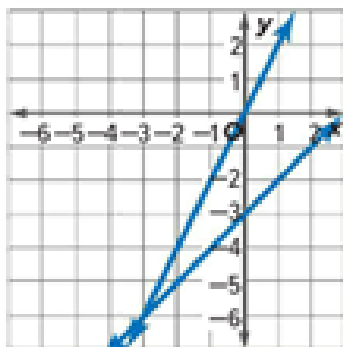
$$x - 3 = 2x$$
$$+x \qquad -x$$
$$-3 = 1x$$
$$-3 = x \quad *$$

$$y = x - 3$$
$$y = -3 - 3 \qquad \text{Solution} \; (-3, -6)$$
$$y = -6 \; *$$

$$x = -3 \; *$$

$$y = 2x$$
$$y = 2(-3) \qquad (-3, -6)$$
$$y = -6 \; *$$

Got it? Do these problems to find out.

Solve each system of equations algebraically.

a. $y = x + 4$ $(-2, 2)$
   $y = 2$

$x + 4 = 2$
$\underline{-4 \quad -4}$
$x = -2$

$y = x + 4$

$y = -2 + 4$

$y = 2$

b. $y = x - 6$ $x - 6 = 3x$
   $y = 3x$ $-x \qquad -x$

$\underline{}$

$\dfrac{-6}{2} = \dfrac{2x}{2}$

$y = 3x \qquad -3 = x$

$y = 3(-3)$ $\underline{}$

$y = -9$ $(-3, -9)$

Slope-Intercept and Standard Forms

Sometimes one or both equations are written in standard form. When solving a system by substitution, one of the equations should be solved for either x or y.

Example

2. Solve the system of equations algebraically.

$y = 3x + 8$

$8x + 4y = 12$

$8x + 4y = 12$	Write the equation.
$8x + 4(3x + 8) = 12$	Replace y with $3x + 8$.
$8x + 4 \cdot 3x + 4 \cdot 8 = 12$	Distributive Property
$8x + 12x + 32 = 12$	Simplify.
$20x + 32 = 12$	Collect like terms.
$20x + 32 = 12$	
$\underline{-32 = -32}$	Subtraction Property of Equality
$20x = -20$	Simplify.
$\dfrac{20x}{20} = \dfrac{-20}{20}$	Division Property of Equality
$x = -1$	Simplify.

Since $x = -1$, replace x with -1 in the equation $y = 3x + 8$ to find the value of y.

$y = 3x + 8$

$y = 3(-1) + 8$ or 5

The solution of this system is $(-1, 5)$.

$y = 3x + 8$

$8x + 4(3x + 8) = 12$

$8x + 12x + 32 = 12$

$20x + 32 = 12$

$x = -1$

$\begin{array}{r} -32 \quad -32 \\ \hline \end{array}$

$\dfrac{20x}{20} = \dfrac{-20}{20}$

$y = 3x + 8$

$y = 3(-1) + 8$

$y = -3 + 8$

$y = 5$

$x = -1$ *

$(-1, 5)$